Chapter E2

Q1 We use $b = \frac{L}{4\pi d^2} \Rightarrow L = b \times 4\pi d^2$ so that
$L = 3.0 \times 10^{-8} \times 4\pi \times (70 \times 9.46 \times 10^{15})^2$ i.e. $L = 1.7 \times 10^{29}$ W.

Q2 We use $b = \frac{L}{4\pi d^2}$ so that $b = \frac{4.5 \times 10^{28}}{4\pi \times (88 \times 9.46 \times 10^{15})^2} = 5.2 \times 10^{-9}$ W m^{-2}.

Q3 From $b = \frac{L}{4\pi d^2}$ we get $d = \sqrt{\frac{L}{4\pi b}}$, i.e. $d = \sqrt{\frac{6.2 \times 10^{32}}{4\pi \times 8.4 \times 10^{-10}}} = 2.4 \times 10^{20}$ m.
This corresponds to $\frac{2.4 \times 10^{20}}{9.46 \times 10^{15}} \approx 26$ kly.

Q4 (a) From $L = \sigma A T^4$, $\frac{L_H}{L_C} = \frac{\sigma A (4T)^4}{\sigma A T^4} = 4^4 = 256$. (b)

$$\frac{b_H}{b_C} = \frac{\frac{L_H}{4\pi d_H^2}}{\frac{L_C}{4\pi d_C^2}} \Rightarrow 1 = \frac{L_H}{L_C} \times \frac{d_C^2}{d_H^2} = 256 \frac{d_C^2}{d_H^2} \text{ and so } \frac{d_C^2}{d_H^2} = \frac{1}{256} \Rightarrow \frac{d_C}{d_H} = \frac{1}{16}$$

Q5 $\frac{b_A}{b_B} = \frac{\frac{L_A}{4\pi d^2}}{\frac{L_B}{4\pi d^2}} \Rightarrow \frac{9.0 \times 10^{-12}}{3.0 \times 10^{-13}} = \frac{L_A}{L_B}$, i.e. $\frac{L_A}{L_B} = 30$.

Q6 Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$: (a) $1 = \frac{R_A^2 (5000)^4}{R_B^2 (10000)^4} \Rightarrow \frac{R_A}{R_B} = \sqrt{\frac{(10000)^4}{(5000)^4}} = 4$. (b)

$$\frac{4.7 \times 10^{27}}{3.9 \times 10^{26}} = \frac{R_{star}^2 (9250)^4}{R_{sun}^2 (6000)^4} \Rightarrow \frac{R_{star}}{R_{sun}} = \sqrt{\frac{4.7 \times 10^{27}}{3.9 \times 10^{26}} \times \frac{(6000)^4}{(9250)^4}} \approx 1.5\,.$$

Q7 Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$: (a)

$$\frac{5.2 \times 10^{28}}{3.9 \times 10^{26}} = \frac{R_{star}^2 (4000)^4}{R_{sun}^2 (6000)^4} \Rightarrow \frac{R_{star}}{R_{sun}} = \sqrt{\frac{5.2 \times 10^{28}}{3.9 \times 10^{26}} \times \frac{(6000)^4}{(4000)^4}} \approx 26\,. \text{ (b) } \frac{b_A}{b_B} = \frac{\frac{L_A}{4\pi d_A^2}}{\frac{L_B}{4\pi d_B^2}} \text{ and}$$

so $2 = \frac{d_B^2}{d_A^2} \Rightarrow \frac{d_A}{d_B} = 0.71$.

Q8 We have that $b = \dfrac{L}{4\pi d^2}$ and $L = \sigma A T^4$. Combining, $b = \dfrac{\sigma A T^4}{4\pi d^2}$. Hence,

$$\frac{b_A}{b_B} = \frac{\dfrac{\sigma A T_A^4}{4\pi d_A^2}}{\dfrac{\sigma A T_B^4}{4\pi d_B^2}} = \frac{\dfrac{T_A^4}{d_A^2}}{\dfrac{T_B^4}{d_B^2}} = \frac{T_A^4 d_B^2}{T_B^4 d_A^2} \Rightarrow \frac{T_A}{T_B} = \sqrt[4]{\frac{b_A}{b_B}\frac{d_A^2}{d_B^2}}$$

. Since this is a ratio we do not have to change units (light years to meters.) Hence, $\dfrac{T_A}{T_B} = \sqrt[4]{\dfrac{8.0\times 10^{-13}}{2.0\times 10^{-15}}\dfrac{120^2}{150^2}} = 4$.

Q9 We know that $L = \sigma A T^4$ and so $\dfrac{L_A}{L_B} = \dfrac{\sigma A_A T_A^4}{\sigma A_B T_B^4}$. Since the radius of A is double that of B, $\dfrac{L_A}{L_B} = 4\times\dfrac{T_A^4}{T_B^4}$. From Wien's law, $\lambda T = const$ and so

$650\times T_A = 480\times T_B \Rightarrow \dfrac{T_A}{T_B} = \dfrac{480}{650}$. Hence, $\dfrac{L_A}{L_B} = 4\times\left(\dfrac{480}{650}\right)^4 = 1.2$.

Q10 The surface temperature determines the color of the star through Wien's law and hence the spectral class.

Q11 The color of the star corresponds to a particular wavelength. This is the peak wavelength in the spectrum which in turn is related to surface temperature through Wien's law, 2.9×10^{-3} K m.

Q12 A star high on the main sequence has high luminosity. The rate at which energy is produced per unit mass is higher and so it will consume its mass in less time, spending less time on the main sequence. (See also the mass – luminosity relation in AHL.)

Q13 The HR diagram is a plot of luminosity (or absolute magnitude) on the vertical axis and temperature (increasing to the left) on the horizontal axis. The horizontal axis may be also be labeled by the spectral class of the star. The diagram shows three major groupings of stars. Main sequence stars which occupy a strip going diagonally down from top left to bottom right. Read giants in the top left part of the diagram and white dwarfs at the bottom left.

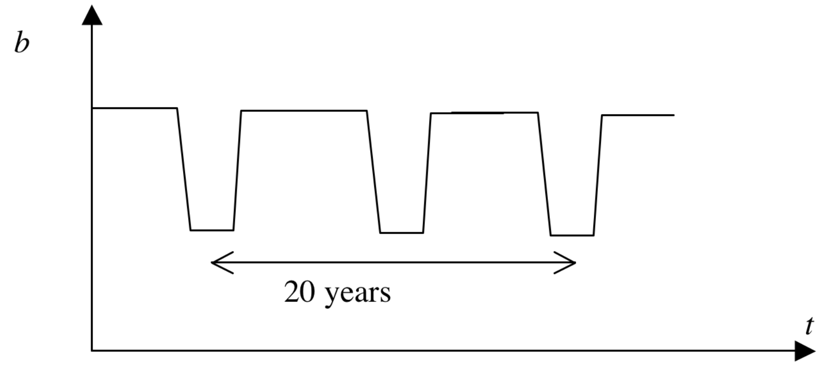
Q14 Light emitted from the star will have to pass through the outer layers of the star. Atoms in these layers may absorb light of certain wavelengths if these wavelengths correspond to energy differences in the atomic energy levels. The absorbed photons will therefore not make it through the outer layers of the star and will appear as dark lines in the spectrum of the star.

Q15 The "line" would be wide, like a bell curve, because of Doppler shifted light from parts of the star approaching and others moving away from the observer on earth.

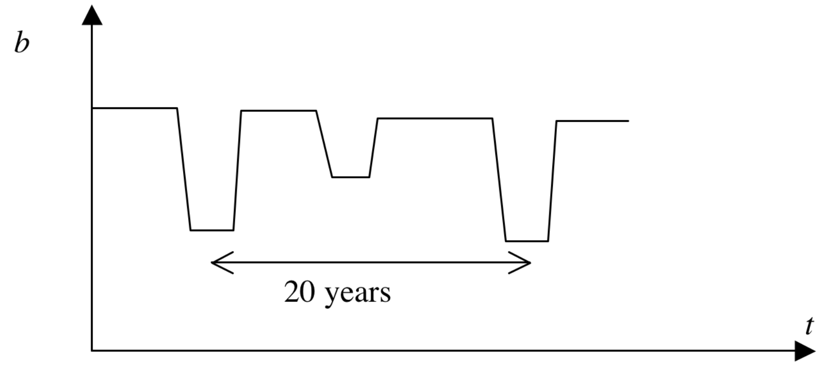
Q16 The common center of the orbits is the center of mass of the two stars. If the stars have equal mass, then the center of mass is half – way between the stars i.e. the orbits have the same radius.

Q17

(a)



(b)



The large decrease in apparent brightness occurs when the brighter star is being blocked.

Both diagrams assume that the size of the two stars is about the same.

Q18 Using the formula (not on the syllabus) $T^2 = \frac{4\pi^2 d^3}{G(M_1 + M_2)}$ we find

$M_1 + M_2 = \frac{4\pi^2 \times (2.4 \times 10^{12})^3}{6.67 \times 10^{-11} \times (40 \times 365 \times 24 \times 3600)^2} = 5.14 \times 10^{30}$ kg . (b) Since

$\frac{M_1}{M_2} = 1.20$ it follows that $M_1 = \frac{1.20}{2.20} \times 5.14 \times 10^{30} = 2.80 \times 10^{30}$ kg and so

$M_2 = 2.34 \times 10^{30}$ kg .

Q19 (a) The distance is $s = d\theta$ where the angular separation is in radians. Hence $s = 5 \times \left(\frac{4.5}{3600}\right)\frac{\pi}{180} = 1.1\times 10^{-4}$ pc $= 1.1\times 10^{-4} \times 3.09\times 10^{16} = 3.4\times 10^{12}$ m. (b) Using the formula (not on the syllabus) $T^2 = \frac{4\pi^2 d^3}{G(M_1 + M_2)}$ we find $M_1 + M_2 = \frac{4\pi^2 \times (3.4\times 10^{12})^3}{6.67\times 10^{-11} \times (87.8\times 365\times 24\times 3600)^2} = 2.95\times 10^{30}$ kg. (c) The radius of one of the orbits is $R = 5\times \left(\frac{1.9}{3600}\right)\frac{\pi}{180} = 1.1\times 10^{-4}$ pc $= 4.61\times 10^{-5} \times 3.09\times 10^{16} = 1.4\times 10^{12}$ m. The radii of the two orbits divide the separation of the stars (3.4×10^{12} m) in the ratio of the masses, i.e. the ratio of the masses is $\frac{1.4\times 10^{12}}{2.0\times 10^{12}} = 0.70$. Hence, $M_1 = \frac{0.70}{1.70}\times 3.0\times 10^{30} = 1.2\times 10^{30}$ kg (outer star) and $M_2 = 1.7\times 10^{30}$ kg (inner star).

Q20 A white dwarf star is an end stage in the evolution of a star. It is very hot, small in size and of small luminosity. It differs from a main sequence star of the same temperature mainly in the mass, and luminosity and radius all of which are very much smaller.

Q21 The density will be $\rho = \frac{1.0\times 10^{30}}{\frac{4\pi}{3}(6.4\times 10^{6})^3} = 9.1\times 10^{8}\ \text{kg m}^{-3}$.

Q22 Initially the luminosity and surface temperature were lower than the values on the main sequence. So it had to be to the right and lower of the present position.

Q23 (a) As the stars rotate, their velocity changes direction. When they move at right angles to the line of sight there is no Doppler shift (first and third diagram). When the faster of the two stars approaches the earth it suffers a blueshift and the slow star a smaller redshift (second diagram). In the fourth diagram, the fast star experiences a large redshift and the slow one a smaller blueshift. (b) The blue and redshifts are not the same in magnitude. This means that the stars have different speeds. In turn this implies that the orbit in different orbits and so the inner star is slower star and the more massive. (c) See answers in textbook.

Q24 Since the redshifts and blueshifts are given by $z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$, the ratio of blushoft to redshift is $\frac{z_1}{z_2} = \frac{v_1}{v_2}$. Since $v^2 = \frac{GM^2}{dM_{total}}$, it follows that $\frac{z_1}{z_2} = \frac{M_1}{M_2} = 0.72$ or $\frac{M_2}{M_1} = 1.4$.

Q25 Since the stars are in a binary system their distances from earth are essentially equal. Hence $\frac{b_1}{b_1} = \frac{\frac{L_A}{4\pi d^2}}{\frac{L_B}{4\pi d^2}} \Rightarrow 10 = \frac{L_1}{L_2}$.

Q26 (a) The peak wavelength is about $\lambda = 0.40 \times 10^{-6}$ m and so the surface temperature (form Wien's law) is $T = \frac{2.9 \times 10^{-3}}{0.40 \times 10^{-6}} \approx 7.2 \times 10^{3}$ K. (b) From the H-R diagram the luminosity is about 5-8 times that of the sun.