Chapter E3

Q1 See discussion in textbook. The method fails for stars far away (more than about 300 pc or 1000 ly) because then the parallax angle is too small to be measured accurately.

Q2 (a) Apparent magnitude is a measure of the brightness of the star, on a logarithmic scale, as it appears from earth. (b) Absolute magnitude is the apparent magnitude a star would have when observed from 10 pc away.

Q3 The distance is $\frac{1}{0.285} = 3.51 \text{ pc}$.

Q4 The distance is $\frac{10.8}{3.26} = 3.31 \text{ pc}$ and so the parallax angle is $\frac{1}{3.31} = 0.302''$.

Q5 (a) The distance is $\frac{1}{0.0067} = 149 \text{ pc}$. (b) The diameter is

$$D = d\theta = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180} \text{ pc} = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180} \times 3.26 \times 9.46 \times 10^{15} = 3.56 \times 10^{11} \text{ m.}$$

And so the radius is $\frac{3.56 \times 10^{11}}{2} = 1.78 \times 10^{11} \text{ m}$. This is about 256 times larger than the radius of the sun.

Q6 The distance is $\frac{1}{0.025} = 40 \text{ pc}$. The actual distance is greater than 10 pc and so the star *appears* dimmer than the equivalent of magnitude 0.8. Hence its apparent magnitude is greater than 0.8. Or, from $m - M = 5\log\frac{d}{10}$ we get

$m = 0.8 + 5\log 4 = 3.8$.

Q7 (a) The distance is $\frac{1}{0.250} = 4.00 \text{ pc}$. (b) It is dimmer than the limit of $m = 6$ and so cannot be seen by the naked eye.

Q8 The stars differ by $\Delta M = 2$ and so have a luminosity ratio of $100^{2/5} = 6.3$.

Q9 The stars differ by $\Delta M = 1.1$ and so have a luminosity ratio of $100^{1.1/5} = 2.8$ with Cappella being the brighter of the two.

Q10 (a) Star A appears brighter because its apparent magnitude is smaller. (b) The distance of star A is larger since its parallax is smaller. Since it appears brighter and it is further away it must have a larger luminosity than star B/

Q11 (a) the luminosity is the same since the absolute magnitude is the same. (b) Star B has a larger parallax so it is closer. Hence it appears brighter.

Q12 Since the stars are part of a binary they have roughly the same distance from us. Hence what appears brighter (star A in this case because of the smaller apparent magnitude) is intrinsically brighter.

Q13 The temperature sis found from

$\lambda T = 2.90\times10^{-3} \Rightarrow T = \dfrac{2.90\times10^{-3}}{2.42\times10^{-7}} \approx 12{,}000\text{ K}$. From the HR diagram a main sequence star at this temperature has a luminosity that is about 100 times larger than the sun's, i.e. $3.9\times10^{28}\text{ W}$. Then,

$$b = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3.9\times10^{28}}{4\pi\times8.56\times10^{-12}}} = 1.9\times10^{19}\text{ m} = 2.0\times10^{3}\text{ ly}.$$

Q14 The distance to the star is $\dfrac{1}{0.034} = 29.4\text{ pc} = 29.4\times3.09\times10^{16} = 9.08\times10^{17}\text{ m}$.

The apparent brightness is then $b = \dfrac{L}{4\pi d^2} = \dfrac{2.45\times10^{28}}{4\pi\times(9.08\times10^{17})^2} = 2.4\times10^{-9}\text{ Wm}^{-2}$.

The easiest way to find the apparent magnitude is to use a formula that is not on the syllabus: $m = -\dfrac{5}{2}\log\dfrac{b}{b_0} = -\dfrac{5}{2}\log\dfrac{2.36\times10^{-9}}{2.52\times10^{-8}} = 2.57$.

But the answer can be found within what is in the syllabus in a somewhat harder way: the absolute magnitude is found from $m - M = 5\log\dfrac{d}{10}$, i.e.

$M = m - 5\log\dfrac{d}{10}$ $M = 2.57 - 5\log\dfrac{29.4}{10} = 0.228$. In bringing the star to a distance of 10 pc its brightness would increase by a factor of $(\dfrac{29.4}{10})^2 = 8.644$. Hence,

$8.644 = 2.512^{\Delta m} \Rightarrow \Delta m = 2.342$. Hence the apparent magnitude is $m = 2.342 + 0.228 = 2.57$.

Q15 (a) We use the non – syllabus formula:

$$\frac{b}{b_0} = 100^{-m/5} = 100^{-1/5} = 0.398 \Rightarrow b = 0.398\times2.52\times10^{-8} = 1.003\times10^{-8} \approx 1.0\times10^{-8}\text{ W m}^{-2}.$$

(b) The ratio in apparent brightness between Procyon and Altair is $\dfrac{1.78}{1.003} = 1.77$ (Procyon being the brighter of the two). Hence,

$1.77 = 2.512^{\Delta m} \Rightarrow \Delta m = 0.6199 \approx 0.62$. Hence the apparent magnitude of Procyon is $1 - 0.62 = 0.38$. Equivalently, we may use the non – syllabus formula:

$$m = -\frac{5}{2}\log\frac{b}{b_0} = -\frac{5}{2}\log\frac{1.78\times10^{-8}}{2.52\times10^{-8}} = 0.38.$$

Q16 Assuming a luminosity of 3500 solar luminosities (the graph is hard to read) we find $b = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3500 \times 3.9 \times 10^{26}}{4\pi \times 3.45 \times 10^{-14}}} = 1.77 \times 10^{21}\,\text{m} = 1.9 \times 10^{5}\,\text{ly}$.

Q17 (a) From $m - M = 5\log\frac{d}{10}$ we have that $0.1 - (-7.0) = 5\log\frac{d}{10}$ and so $0.1 - (-7.0) = 5\log\frac{d}{10} \Rightarrow d = 10 \times 10^{\frac{7.1}{5}} = 263 \approx 260$ pc. (b) The distance is $\frac{1}{0.760} = 1.32$ pc. Hence $(-0.27) - M = 5\log\frac{1.32}{10}$ and so $M = 4.13 \approx 4.1$. (c) We present two approaches. First we can find the absolute magnitude using the distance to the sun $1.0\text{ AU} = 4.86 \times 10^{-6}$ pc as $(-26.74) - M = 5\log\frac{4.86 \times 10^{-6}}{10}$ i.e. $M = 4.83$. Then the distance at which the apparent magnitude becomes the limiting $m = 6.0$ is $6.0 - 4.83 = 5\log\frac{d}{10} \Rightarrow d = 10 \times 10^{\frac{1.17}{5}} \approx 17$ pc. Alternatively, we can see the sun when the apparent magnitude does not exceed the limit of $m = 6.0$. Then $\Delta m = 32.74$ and so at the larger distance the apparent brightness will drop by $100^{32.74/5} = 1.25 \times 10^{13}$. The distance will then be larger than the actual sun distance by a factor of $\sqrt{1.25 \times 10^{13}} = 3.5 \times 10^{6}$, i.e. at a distance of 3.5×10^{6} AU or about 17 pc.