Chapter J2

Q1 (a) Particles are produced in collisions – one example out of many is: a collision of an electron with a positron in a synchrotron. If we produce a pair of a particle and its antiparticle each of mass M we will need an energy of at least $2Mc^2$. This energy will be provided by the electron positron pair. Their total energy will be $2(m_e c^2 + E_K)$. Hence $2(m_e c^2 + E_K) = 2Mc^2 \Rightarrow E_K = Mc^2 - m_e c^2$. Thus, is M is to be large the kinetic energy must large as well. (b) Using the result in (a), $E_K = Mc^2 - m_e c^2 = 175\,\text{GeV} - 0.511\,\text{MeV} \approx 175\,\text{GeV}$. This is the kinetic energy of the electron and we need an equal amount for the positron for a total of 350 GeV.

Q2 To "see" something one must use a wavelength that is at least of the same order of magnitude as the size that is being investigated. If the wavelength is too large the object cannot be seen. Therefore photons and other particles used to probe a structure must have a de Broglie wavelength that is of the same size as the structure itself. For elementary particles of sizes smaller than nuclear diameters (10^{-15} m) we therefore need very small wavelengths. These necessarily imply high energies since $\lambda = \frac{hc}{E}$ for photons and $\lambda = \frac{h}{\sqrt{2mE}}$ for particles with mass.

Q3 (a) The de Broglie wavelength is $\lambda = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3\times10^{8}}{50\times10^{9}\times1.6\times10^{-19}} = 2.5\times10^{-15}\,\text{m}$. (b) (i) Yes since it is comparable to a nucleus diameter. (ii) No since it is larger than a nucleon size.

Q4 The de Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_K}}$. The alpha particle has the larger mass and so the smaller de Broglie wavelength, so it would be more suitable.

Q5 (a) Synchrotron radiation is electromagnetic radiation emitted by a charged particle that is accelerating. (b) This radiation is a constraint on the accelerator because a fraction of the energy provided for acceleration is actually lost as radiation. (c) Synchrotron radiation is more pronounced in circular machines.

Q6 At the same energy the lighter particle would radiate more. In a linac, the length would be much larger to achieve acceleration of a heavy particle such as the proton to energies comparable to those achieved by electrons.

Q7 Neutrons being electrically neutral cannot be accelerated by electric fields.

Q8 The question has a typo! It should say "Why can magnetic field **not** be used to accelerate particles".

(a) Magnetic fields produce a force that is at right angles to the velocity of the particle and so this force does zero work. Hence it cannot change the kinetic energy and hence speed of the particle. (b) They are used to deflect the particles into circular paths.

Q9 One reason is to provide shielding for the synchrotron radiation and another is to be far from vibrations from above.

Q10 With a constant alternating source frequency, the time taken for the voltage in the next tube to change sign is constant. Hence the particle spends the same time traveling in any one tube. However, as the particle moves down the line it is moving faster because it is being accelerated and so the length of the tubes must be increasing. Eventually the particles are moving close to the speed of light and so the speed is practically constant and therefore so is the length of the tubes.

Q11 In a cyclotron, the particle to be accelerated is placed at the center of an area in between opposite poles of a magnet. The particles are emitted with a small initial speed from the center and then begin to follow a circular path inside the region of magnetic field. The lower magnet is split into two parts (the D's) and an alternating potential difference is applied between the two split D's. Every time a particle crosses the D's it is accelerated. For this to happen the frequency of the applied potential difference must equal the frequency of revolution of the particle in the field. This frequency is constant and does not depend on the speed of the particle. With increasing speed, the particle follows a path of increasing radius (a spiral path) and eventually emerges out of the region of the magnetic field with the help of additional magnetic fields.

Q12 The proton increases its energy by 30 keV every time it crosses the D's. To achieve a kinetic energy of 25 MeV it must cross $\frac{25\times 10^{3}}{30}\approx 833$ times and this corresponds to about $\frac{833}{2}\approx 417$ revolutions.

Q13 (a) From the textbook, we know that the cyclotron frequency has to be $f=\frac{qB}{2\pi m}=\frac{1.6\times 10^{-19}\times 1.2}{2\pi\times 1.67\times 10^{-27}}=1.8\times 10^{7}\ \text{Hz}$. (b) The maximum kinetic energy is $E_K=\frac{q^2B^2R^2}{2m}=\frac{(1.6\times 10^{-19}\times 1.2\times 0.20)^2}{2\times 1.67\times 10^{-27}}=4.41\times 10^{-13}\ \text{J}=2.8\ \text{MeV}$. (c) The proton increases its energy by 35 keV every time it crosses the D's. To achieve a kinetic energy of 2.8 MeV it must cross $\frac{2.8\times 10^{3}}{35}=80$ times and this corresponds to $\frac{80}{2}=40$ revolutions.

Q14 The potential difference affects the number of revolutions the particle will make in the region of magnetic field. As shown in the text, the final energy of the particle is determined by how large the radius of the cyclotron is.

Q15 The magnets are used to bend the charged particles in the beam into a circular path.

The radius of the circular path in a magnetic field is given by $R = \frac{mv}{qB}$. As the speed increases the radius would increase as well. However, in a synchrotron the particles follow a circular path of fixed radius. This can happen only if the strength of the magnets increases in such a way so as to keep the radius fixed.

The particles are accelerated by electric fields in between the magnets.

Q16 From $R = \frac{mv}{qB}$, we see that decreasing the B field by factor of 4 would increase the radius by a factor of 4, i.e. the circumference would have to be $4 \times 27 = 108\,\text{km}$.

Q17 (a) The separation of two consecutive bunches is $\frac{27 \times 10^3}{2 \times 3000} = 4.5\,\text{m}$. (b) The bunches have a relative speed of twice the speed of light and so the time for two to meet is $\frac{4.5}{2 \times 3 \times 10^8} = 7.5 \times 10^{-9}\,\text{s}$. (c) The frequency is just the inverse of the meeting time i.e. $f = \frac{1}{7.5 \times 10^{-9}} = 1.33 \times 10^8\ \text{Hz}$. (d) The number of collisions per second is $N = 20f = 2.7 \times 10^9\,\text{s}^{-1}$.

Q18 (a) $qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB} = \frac{p}{qB} = \frac{E}{qBc}$.

(b) $B = \frac{E}{qRc} = \frac{7.0 \times 10^{12} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19} \times 4.26 \times 10^3 \times 3 \times 10^8} = 5.5\,\text{T}$.

Q19 The great advantage of the synchrotron is that much higher energies can be achieved. The linear accelerator would require a very great length to achieve a comparable energy. In a synchrotron, all of the total energy of the colliding particles is available energy to produce the rest energy of new particles. In a linac the target is stationary and some of the energy must be provided to give kinetic energy to the products in addition to rest energy. Another advantage is that the time of the collision can be controlled (unlike the situation in a linac).

Q20 In a colliding beam experiment all the energy of the colliding particles can go into producing new particles. In a stationary target experiment, the energy must go into producing the particles as well as providing them with sufficient kinetic energy to satisfy momentum conservation. In the colliding beam, no kinetic energy needs to be provided to the created particles because the total momentum before the collision is zero.

Q21 The disadvantage of colliding beam experiments is the low probability of collision and the very large percentage of energy lost as synchrotron radiation.

Q22 The total energy is $2\times 938 = 1876\,\text{MeV}$. Each photon must carry away half of this energy i.e. 938 MeV. Hence, $\frac{hc}{\lambda} = 938\,\text{MeV} = 938\times 10^{6}\times 1.6\times 10^{-19}\,\text{J}$, i.e.

$$\lambda = \frac{6.63\times 10^{-34}\times 3\times 10^{8}}{938\times 10^{6}\times 1.6\times 10^{-19}} = 1.3\times 10^{-15}\,\text{m}.$$

Q23 We use $E_A^2 = 2mc^2E + 2(mc^2)^2$ where m is the mass of the target (the proton). Here, $E_A = 938 + 940 + 140 = 2018\,\text{MeV}$. Hence
$2018^2 = 2\times 938\times E + 2\times 938^2 \Rightarrow E = 1233\,\text{MeV}$. The kinetic energy of the proton is then $K = 1233 - 938 = 295\,\text{MeV}$. (The answer in the textbook gives the total energy of the proton.)

Q24 We use $E_A^2 = 2mc^2E + 2(mc^2)^2$. The available energy must be $E_A = 4mc^2$ and so $16(mc^2)^2 = 2mc^2E + 2(mc^2)^2 \Rightarrow 2mc^2E = 14(mc^2)^2$, i.e. $E = 7mc^2$.

Q25 The energy required to produce the particles is $E_A = 2\times 938 + 135 = 2011\,\text{MeV}$. Hence the total energy of each of the protons must be $E = \frac{2011}{2} = 1005.5\,\text{MeV}$. The kinetic energy is then $E_K = 1005.5 - 938 = 67.5\,\text{MeV}$.

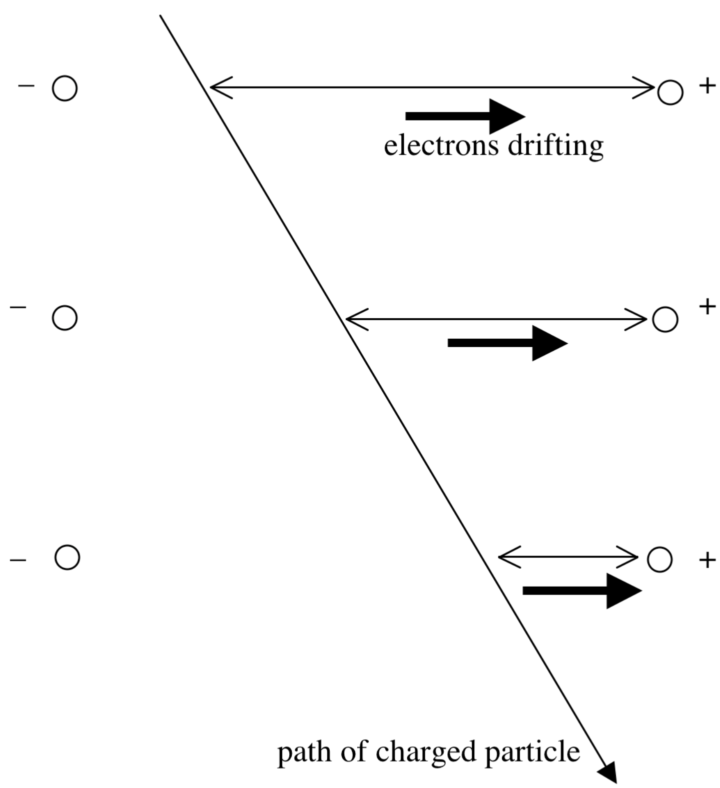
Q26 We use $E_A^2 = 2Mc^2E + (Mc^2)^2 + (mc^2)^2$ where M is the mass of the target (the proton). Here, $E_A = 1193 + 498 = 1691\,\text{MeV}$. Hence
$1691^2 = 2\times 938\times E + 938^2 + 140^2 \Rightarrow E = 1045\,\text{MeV}$.

Q27 Detectors are placed around the collision point in a layered fashion. They begin with a drift chamber that will determine the paths of particles. This is followed by calorimeters that measure the energy of the particles and end with muon detectors. The first thing that the experimenter wants to know is the direction of motion of the produced particles. This is done in the drift chambers which is why they are put closest to the interaction point. It is important that in the drift chamber (see Q30) the electric fields do not change the kinetic energy and the direction of the produced particles.

Q28 They measure the energy of a particle. They are typically sheets of lead or iron with gas in between. The particle will cause the emission of other particles as it enters the calorimeters. In the electromagnetic calorimeter ionization caused by electrons, positrons and photons in the gas inside the calorimeter is detected. The amount of ionization is related to the energy of the particle. Hadrons pass through the electromagnetic calorimeter relatively unaffected and enter the thicker hadronic calorimeter that surrounds the electromagnetic calorimeter. Its construction is similar to the electromagnetic calorimeter – the difference is that in the electromagnetic calorimeter it is the electromagnetic force that is responsible for creating the secondary particles whereas in the hadronic, it is the strong interaction. Muons, being very penetrating, pass through the electromagnetic and hadronic calorimeters relatively unaffected and so the muon detectors are placed in the outer layers.

Q29 In a bubble chamber, a liquid (usually this is liquid hydrogen) is kept near its boiling point. A charged particle entering the chamber will produce ions along its path and bubbles will be formed along the path. This is photographed giving an image of the path. A magnetic field bends the paths of the particles and the radius of the path gives information on the momentum of the particle.

Q30 Consider the diagram below, which shows six fine wires (represented by the circles). The wires are into the plane of the page. The wires on the left are kept at a negative potential and those on the right at a positive potential. Therefore there is an electric field between the wires. These wires are in a chamber and the chamber is filled with a gas.



A charged particle enters the chamber along the path shown. Assume for simplicity that the path is along the plane of the page.

The particle will ionize the gas and electrons will drift towards the wires with the positive potential. By suitable choice of gas and wire electric potentials *the electron drift speed can be made constant* (or almost constant). The arrival of the electrons at each wire is registered as a small current and the arrival time is recorded (the uncertainty in the arrival time is less than 1 ns). Since the drift speed is constant and known, we *can deduce the distances the electrons traveled to get to the wire*.

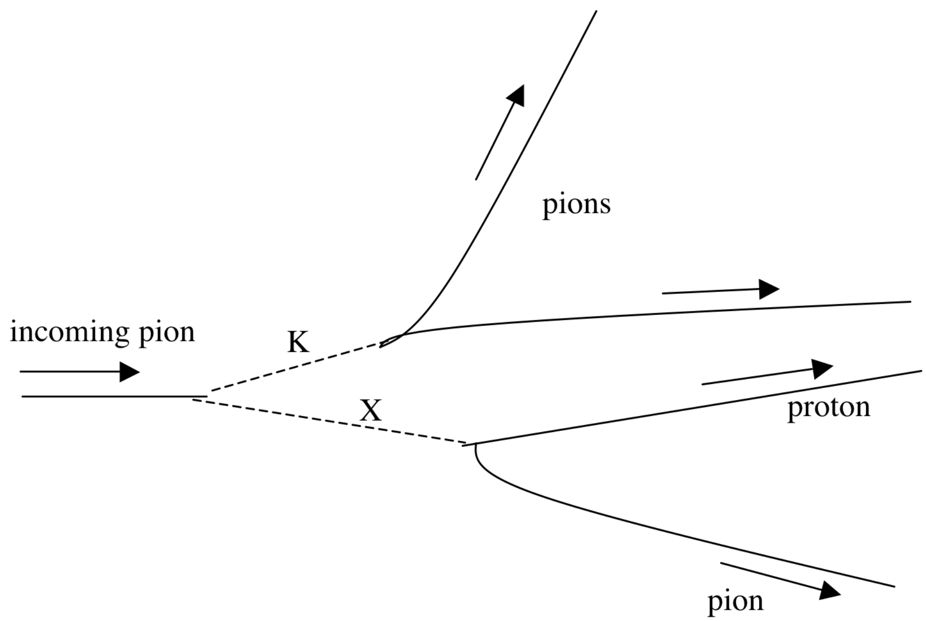
To fully reconstruct the path, we need an additional set of wires at right angles to the first set. The two sets of wires together (a *grid*) can be used to find the position from which the electrons were emitted, i.e. the position of the charged particle on a plane. A second grid of wires parallel to the first can then be used to locate the position of the particle in three dimensions.

Q31 Drift chambers have two main advantages over the bubble chamber. The first is that no photographs are taken. The images are digital and so are analyzed by a computer. With bubble chamber photographs thousands of hours were spent analyzing the images. The second advantage is that there is no "dead" time: with the bubble chamber a certain amount of time had to pass from taking one photograph to the next.

Q32 The question is admittedly vague – apologies! The idea is to use the principles of energy and momentum conservation to the process that leads to the creation of a particle.

We can make the question more concrete *and only for those taking the relativity option* by asking the following question. You will need to use the relativistic formula $E^2 = (mc^2)^2 + p^2c^2$. This is a homework exercise for the good physics student – something like this will not appear on an IB exam. It also assumes knowledge of chapter J3.

New question 32 The diagram is a copy of part of a bubble chamber photograph showing a pion (π^-) colliding with a stationary proton and producing a neutral kaon and an unknown particle X: $p + \pi^- \to K^0 + X$. The kaon subsequently decays into two pions and particle X decays in a proton and a pion. The dashed lines showing the paths of the K and the X are not part of the photograph and are inferred by the experimenter.



(a) Explain why the proton that was bombarded by the pion does not show in the photograph. (b) Explain why the paths of the K and the X also did not show in the photograph. (c) Explain whether the decay of the kaon is given by $K^0 \to \pi^- + \pi^+$ or $K^0 \to \pi^0 + \pi^0$. (d) The incoming pion had a momentum of $920\,\mathrm{MeV\,c^{-1}}$. The kaon is produced with momentum $240\,\mathrm{MeV\,c^{-1}}$ at an angle of 12° to the direction of the pion. Calculate the mass of the unknown particle X. (e) Particle X is a hadron. Is it a meson or a baryon?

(You are given the masses: proton: $938\,\mathrm{MeV\,c^{-2}}$, pion: $135\,\mathrm{MeV\,c^{-2}}$ and kaon: $498\,\mathrm{MeV\,c^{-2}}$.)

Answer (a) Because it is stationary and so causes no ionization. (b) Because they are neutral and so cause no ionization. (c) It is a decay into charged pions, $K^0 \to \pi^- + \pi^+$. The tracks of the neutral pions would not show up in the photograph. Or, we see that the pions are turning in opposite directions in the magnetic field of the bubble chamber. This implies that they are oppositely charged. (d) Applying momentum conservation

$$920 = 240\cos 12^\circ + p_x \quad \Rightarrow p_x = 685\,\mathrm{MeV\,c^{-1}}$$
$$0 = 240\sin 12^\circ - p_y \quad \Rightarrow p_x = 50\,\mathrm{MeV\,c^{-1}}$$

The momentum of the X is therefore $p_X = \sqrt{p_x^2 + p_y^2} = \sqrt{685^2 + 50^2} = 687\,\mathrm{MeV\,c^{-1}}$. The total energy before the collision is $m_p c^2 + \sqrt{(m_\pi c^2)^2 + p_\pi^2 c^2} = 938 + \sqrt{135^2 + 920^2} = 1868\,\mathrm{MeV}$. The total energy of the produced kaon is $\sqrt{(m_K c^2)^2 + p_K^2 c^2} = \sqrt{498^2 + 240^2} = 553\,\mathrm{MeV}$ and so the total energy

of the X is $E = 1868 - 553 = 1315\,\text{MeV}$. Hence the rest energy of the X is $m_X c^2 = \sqrt{E^2 - p_X^2 c^2} = \sqrt{1315^2 - 687^2} = 1120\,\text{MeV}$. The mass is $m_X = 1120\,\text{MeV c}^{-2}$.
(e) To conserve baryon number, X has to be a baryon. (The X is in fact the Λ^0 baryon).

Q33 This is for you! You may want to consider the actual cost per capita for this research and then compare it to the cost per capita for military expenditures of various countries. You may also want to refer to the importance of nuclear magnetic resonance imaging and PET scans in medicine and how these techniques originated in nuclear and particle physics.